Absolute vs. Relative Notion of Wealth Changes

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Abstract

This paper discusses solutions derived from lottery experiments using two alternative assumptions: that people perceive wealth changes as absolute amounts of money; and that people consider wealth changes as a proportion of some reference value dependant on the context of the problem under consideration. The former assumption leads to the design of Prospect Theory, the latter - to a solution closely resembling the utility function hypothesized by Markowitz (1952B). This paper presents several crucial arguments for the latter approach. That essentially all financial theories consider "returns" expressed in relative terms, rather than "gains" and "losses" expressed as monetary amounts, is one of them. This provides arguments for rejecting the Prospect Theory paradigm.

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1. Introduction

Prospect Theory (Kahneman, Tversky, 1979), and its Cumulative version (Tversky, Kahneman, 1992) assert that people are more concerned with changes in wealth than its overall value when making decisions involving small sums of money. Anticipated gains and losses, expressed as monetary amounts, are then used to evaluate the prospects under consideration. An analysis of lottery experiments assuming the absolute notion of wealth changes requires, however, the concept of probability weighting to be incorporated into the descriptive model.

The present paper questions the assumption that people treat gains and losses as absolute values when making decisions under conditions of risk. On the contrary, it asserts that gains and losses are perceived in relative terms, in a context which depends on how attention

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is focused on it. The result is that gains and losses are perceived in relation to a reference value which, most frequently, is the maximum prospect outcome. Analyzing gains and losses in relative terms eliminates the need for the probability distortion concept and leads to a solution closely resembling the utility function hypothesized by Markowitz (1952B).

This paper presents several important arguments in support of the latter approach, the major one being that people regard changes in wealth in relative terms is founded on Weber's Law - one of the fundamental laws of psychophysics. This law contradicts the absolute notion of wealth changes. The observation is not new and has been confirmed by modern researchers including Kahneman, Tversky (1984) and Thaler (1985, 1999) since the introduction of Prospect Theory. Moreover, "returns" expressed as percentages of the investment, rather than "gains" and "losses" expressed as monetary amounts, are generally used in finance. This offers a rationale for repudiating the Prospect Theory approach.

2. Absolute Notion of Wealth Changes

Consider the following set of experiments[2]:

Experiment 1: *Would you prefer to enter a lottery with a 50% chance of winning either $100 or $0, or to receive a payment of $50?* Most respondents (59.1%) would prefer to receive the certain payment.

Experiment 2: *Would you prefer to enter a lottery with a 5% chance of winning $1000 and $0 otherwise, or to receive a payment of $50?* Most respondents (53.5%) would prefer the lottery.

Interestingly, the respondents exhibit risk aversion behavior in Experiment 1 and risk seeking behavior in Experiment 2, for the same certain payment amount of $50. It may be concluded that people are generally risk averse (as demonstrated in Experiment 1), but there has to exist an additional effect related to the perception of probabilities in order to explain the risk-seeking behavior observed in Experiment 2. This effect would have to rely on a non-linear perception of probabilities and especially on an overweighting of low ones. This reasoning thus leads to a theory in which there exists a non-linear probability weighting function in addition to a utility function. This is how Prospect Theory (and other theories using a similar approach) developed.

[2] The four experiments discussed correspond with the well-known "fourfold pattern" of risk attitudes stated by Prospect Theory and its cumulative version (Kahneman, Tversky, 1979, 1992). Anyhow, an Internet survey has been conducted in order to support the reasoning presented, especially that Kahneman and Tversky used different outcome and probability values. The experiment design is briefly described in Appendix 1, as it is not essential to this paper. The experiment results are given in parentheses.

Consider the following further set of experiments:

Experiment 3: *Would you prefer to enter a lottery with a 50% chance of losing $100 and $0 otherwise, or to pay $50 to avoid the game?* Most respondents (56.2%) would prefer the lottery.

Experiment 4: *Would you prefer to enter a lottery with a 5% chance of losing $1000 and $0 otherwise, or to pay $50 to avoid the game?* Most respondents (63.4%) would prefer to pay the certain amount.

The respondents in Experiment 3 exhibit risk seeking behavior, whereas the respondents in Experiment 4 exhibit risk aversion behavior for exactly the same certain payment amount of $50. As in case of gains, a probability weighting function needs to be incorporated into the descriptive model to explain the pattern observed. Experiment 3 leads to the conclusion that people are generally risk *seeking* for losses, and a probability weighting function explains the risk aversion observed in Experiment 4.

3. Relative Notion of Wealth Changes

The foregoing reasoning assumed the absolute notion of wealth changes. This means that gains and losses were represented and analyzed as monetary amounts (such as $50). This is one of the basic assumptions of Prospect Theory and is best expressed by a value function which supposedly determines the value (utility) of specific amounts of money to people (according to Cumulative Prospect Theory the value function is defined as $v(x) = \lambda |x|^{\alpha}$, where x is the gain or loss expressed as an absolute monetary amount).

People, however, typically consider wealth changes in relative terms. This means that gains and losses are usually perceived as a proportion of a reference value, which depends on the context of the problem. This observation is not new and was also noticed by Kahneman and Tversky in 1984, i.e. 5 years after the introduction of Prospect Theory: *The topical organization of mental accounts leads people to evaluate gains and losses in <u>relative</u> rather than in absolute terms* (emphasis added)[3]. Despite this, the absolute notion of gains and losses remained the underlying assumption of Cumulative Prospect Theory, which appeared in 1992.

The explanation that people regard changes of wealth in relative terms is founded on basic psychophysical laws. In the first half of the nineteenth century, German researcher Ernst

[3] Kahneman and Tversky consider a minimal, topical, and comprehensive account in their "Choices, Values, and Frames" paper. They state: *A topical account relates the consequences of possible choices to a reference level that is determined by the context within which the decision arise* and conclude: *People will spontaneously frame decisions in terms of topical accounts.* The idea of mental accounts actually originated with Thaler (1980, 1999).

Weber conducted experiments on determining the Just Noticeable Difference in weight between objects and concluded that this difference is twice as great with a 2 kg object than with a 1 kg object. The law Weber formulated in 1834 states that this difference is a constant proportion of the initial stimulus magnitude. This is now one of the fundamental laws of psychophysics (Encyclopædia Britannica, 2009):

$$JND / S = k \tag{3.1}$$

JND denotes the Just Noticeable Difference, *S* denotes the initial stimulus magnitude and *k* is a constant. This law holds with reasonable accuracy for most stimuli within a broad range. For example, the value of *k* is 2% for weight, 4.8% for loudness and 7.9% for brightness. It follows from the Weber law that the same change in stimulus (for instance 0.2 kg) can be strongly felt, slightly noticed or not perceived at all depending on the magnitude of the initial stimulus. It further follows that an unambiguous and absolute perception level of a specific stimulus change cannot be determined, as this depends on the situational context.

How the Weber Law works for financial stimuli will be presented with the following example:

Problem 1a: *What is the smallest significant amount of money to a person shopping for goods worth about $100?*

This will possibly be close to 1 dollar (but not as small as 1 cent). Such a person may, for example, consider choosing a rival product that is $0.5 cheaper.

Problem 1b: *What is the smallest amount of money considered by the same person purchasing a house for $500,000?*

This will probably be at least $1,000. A purchase offer of $479,538 is hard to imagine; $480,000 seems far more likely. It follows that ten dollars, a significant amount in the former case, is completely insignificant in the latter. Even $100, the sum total of a person's expenditure in a shop, is of no significance in a house purchase.

The foregoing example demonstrates that the human mental system adapts itself to financial quantities, just as its sensory system does to physical ones. The result is that the Just Noticeable Difference remains an approximately constant proportion of different financial amounts. This means that when considering financial prospects (projects, investments, lotteries etc.), the size of the prospect becomes a reference value in the entire mental process, rendering an absolute amount of money (say $10) relevant or irrelevant depending on the context. This conclusion constitutes a fundamental deviation from Prospect Theory, which regards gains and losses in absolute terms, and attempts to derive a value function in terms of absolute monetary amounts.

The relative perception of outcomes has been documented in real situations. For instance Baltussen, Post and Van den Assem (2008) used an extensive sample of choices from ten different editions of the high stakes TV game show "*Deal or No Deal*": *"In each sample, contestants respond in a similar way to the stakes relative to their initial level, even though the initial level differs widely across the various editions. Amounts therefore appear to be primarily evaluated relative to a subjective frame of reference rather than in terms of their absolute monetary value"*.

Considering outcomes expressed in relative terms is common for essentially all financial theories. For instance Portfolio Theory (Markowitz, 1952A) considers expected returns and their variances, where expected returns are clearly expressed as percentages of the initial investment. A Portfolio Theory that considered gains and losses expressed as monetary amounts would be hard to imagine. This would make no sense whatsoever as a specific amount of money (say $1000) has a different utility value to different individual and institutional investors. Therefore it is "returns" that are generally considered in finance, and not "gains" and "losses" as Prospect Theory posits.

4. Solution Resulting From Relative Notion of Wealth Changes

As presented in Section 2, the consideration of gains and losses in absolute terms inevitably leads to the concept of probability weighting. Considering gains and losses as relative values allows, however, the experimental results to be explained without this concept.

It is assumed that people perceive outcomes as proportions of the main payment. In Experiment 1 the amount of the certain payment is half that of the main payment. The conclusion from this experiment may be that people are risk-averse for the relative outcome of $r = 0.5$. Similarly, Experiment 2 shows that people exhibit a risk-seeking attitude by their preference for the lottery over a relative outcome of $r = 0.05$. Experiment 3 leads to the conclusion that people are risk seeking for the *relative* loss of $r = 0.5$, whereas Experiment 4 leads to the conclusion that people are risk averse for the *relative* loss of $r = 0.05$.

This line of reasoning leads to the plotting of a utility function expressed for *relative* outcome values, which is partially convex and partially concave. This would certainly be a completely different solution from that proposed by Prospect Theory, all the more so since it would not utilize any probability weighting function. This solution is presented in Figure1.

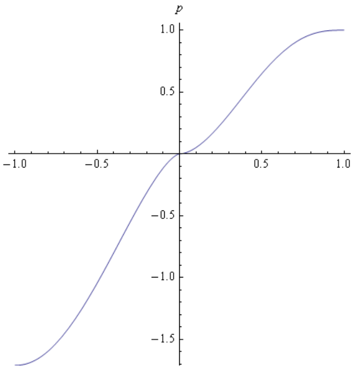

Figure 1. The utility function of relatively expressed outcomes.

The curve should be concave for high relative gains, reflecting the risk aversion observed in Experiment 1, and convex for low relative gains, reflecting the risk seeking attitude observed in Experiment 2. The picture is reversed for losses. The function should be convex for high relative losses, reflecting the risk seeking attitude observed in Experiment 3, and concave for low relative losses, reflecting the risk aversion observed in Experiment 4. The curve should be of greater magnitude for losses than for gains as people are generally averse to loss.

The double-S shape of this hypothetical relative utility function offers a simple and concise explanation of risk-seeking and risk-aversion attitudes when making decisions under conditions of risk. This replaces the “fourfold pattern” of risk attitudes formulated by Cumulative Prospect Theory. This solution does not require a probability weighting function to describe the pattern.

Quite surprisingly, the obtained curve strongly resembles the utility function hypothesized by Markowitz (1952) and shown in Figure 2.

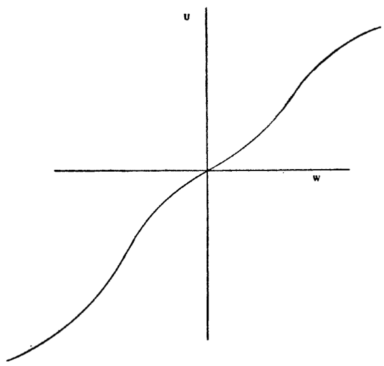

Figure 2. The shape of the utility function according to the Markowitz hypothesis of 1952.

The only significant difference with the Markowitz curve is that the relative utility function is defined for relative values of outcomes rather than for absolute ones.

5. Basic Model

The Continuity axiom is one of the axioms postulated by von Neumann and Morgenstern (1944). This assumes that for lotteries *A*, *B*, and *C* such that $A < B < C$, there exists a probability *p* such that *B* is equally good as $(1-p)A + pC$. Assuming that lotteries *A*, *B*, *C* are simply monetary outcomes, the axiom can be presented in the following form:

$$(1-p)u(P_{min}) + pu(P_{max}) = u(ce) \tag{5.1}$$

where *u* denotes the utility function, P_{min} and P_{max} denote the minimum and maximum lottery outcomes, and *ce* denotes the certainty equivalent. This is the general formula for experimentally deriving the utility function in Expected and Non-Expected Utility Theories. Von Neumann and Morgenstern argued that "*utility is a number up to a linear transformation*". Therefore $u(P_{min}) = 0$ and $u(P_{max}) = 1$ can arbitrarily be assumed, in which case (5.1) simplifies to:

$$p = u(ce) \tag{5.2}$$

This striking result shows that utility is expressed in terms of probability. The greater the probability, the greater the certainty equivalent, and thereby the greater the lottery utility.

It is next noted that the certainty equivalent assumes values in the $[P_{min}, P_{max}]$ range. This allows the relative certainty equivalent *r* to be described using the following transformation:

$$r = \frac{ce - P_{min}}{P_{max} - P_{min}} \tag{5.3}$$

This further allows (5.2) to be presented as:

$$p = D(r) \tag{5.4}$$

where *D* denotes decision utility defined in the [0, 1] range. Decision utility (5.4) describes the utility function presented in Figure 1 separately for gains and losses. This is where the axiomatic approach meets the psychological one presented in Section 3.

To summarize, the model considers outcomes expressed in relative terms, and utility is expressed in terms of probability. This approach is obviously very different from that proposed by Prospect Theory, especially given that it does not have recourse to the concept of probability weighting.

6. Discussion

It is not the purpose of this paper to present the entire decision utility model. This is done elsewhere (Kontek, 2011). Instead, this paper focuses on considerations regarding the absolute versus the relative notion of wealth changes which underlie the model. Several important advantages of the latter approach have been presented in the paper. Most importantly, people do consider changes of wealth in relative terms. This was even confirmed by Kahneman and Tversky – the authors of Prospect Theory.

Applying the relative notion of wealth changes to analyze the experimental data leads to a simple solution strongly resembling the utility function hypothesized by Markowitz (1952B). It appears that the concept of probability weighting – one of the key planks of Prospect Theory – becomes redundant when gains and losses are considered in relative terms. This paper presents an explanation of this conclusion. Prospect Theory proceeds from the assumption that gains and losses are perceived in absolute terms. This, however, inevitably leads to the probability weighting concept in order to explain simple lottery experiments. Once the absolute notion is replaced by the relative one, the need for probability weighting disappears.

This paper strictly demarcates the two approaches as the relative and absolute notions are mutually exclusive. As a result Prospect Theory (Kahneman & Tversky, 1979, 1992) with its probability weighting concept cannot be held to be an accurate explanation of people's behavior while gains and losses are simultaneously held to be perceived in relative terms (Kahneman & Tversky, 1984).

Importantly, changes in wealth are commonly perceived in relative terms in finance. All financial theories consider "returns", rather than "gains" or "losses" expressed as monetary amounts. This supports the model presented here and makes it easier to apply than Prospect Theory.

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Appendix 1.

The Internet experiment was designed using Michael Birnbaum's SurveyWiz tools (Birnbaum, 2000). I became acquainted with these tools during a workshop on Web-Research techniques held at California State University, Fullerton, CA in January 2011. I would like to thank Michael Birnbaum (CSU, Fullerton, CA) and Ulf-Dietrich Reips (University of Deusto, Bilbao, Spain) for the inspiring ideas they presented.

The questionnaire consisted of 12 choices between lotteries and sure payments. The survey was addressed to residents of Poland and so the outcome amounts were expressed in PLN ($1 ≈ 2.8 PLN; the purchasing power, especially for commodities, is often similar). However, the values presented in the paper are given in $ to make the text more readable for international readers. The participants were recruited among members of the psychological association at SWPS University, Wrocław, Poland, and among media people on Facebook. I would like to thank Jakub Traczyk and Małgorzata Błachnio for their efforts in recruiting the respondents. Altogether 45 people responded: 21 male; 21 female; and 3 unknown. Their ages varied from 20 to 83 with a median of 30.

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